PHILIPPINE SCIENCE HIGH SCHOOL – MAIN CAMPUS

Agham Road, Diliman, Quezon City 1101 Philippines

Mathematics 5: Analytic Geometry & Differential Calculus

1st Quarter School Year 2024-2025

Problem Set: Hyperbola and Unified Treatment of Conic Sections

I. Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.

1. $\frac{y^2}{25} - \frac{x^2}{9} = 1$
2. $\frac{x^2}{36} - \frac{y^2}{64} = 1$
3. $4x^2 - y^2 - 24x - 4y + 28 = 0$
4. $y^2 - 4x^2 - 2y + 16x = 31$
5. $x^2 - y^2 + 2y = 1$
6. $9y^2 - 4x^2 - 36y - 8x = 4$

II. Identify the type of conic section whose equation is given and find its vertices and foci.

1. $4x^2 = y^2 + 4$
2. $4x^2 = y + 4$
3. $x^2 = 4y - 2y^2$
4. $y^2 - 2 = x^2 - 2x$
5. $3x^2 - 6x - 2y = 1$
6. $x^2 - 2x + 2y^2 - 8y + 7 = 0$

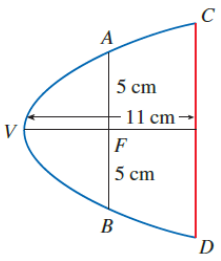
III. Find an equation for the hyperbola that satisfies the given conditions.

1. V: (±3, 0), F: (±5, 0)
2. V: (0, ±2), F: (0, ±5)
3. V: (-3, -4), (-3, 6), F: (-3, -7), (-3, 9)
4. V: (-1, 2), (7, 2), F: (-2, 2), (8, 2)
5. V: (±3, 0), A: y = ±2x
6. F: (2, 0), (2, 8), A: y = 3 + ½ x, y = 5 – ½ x

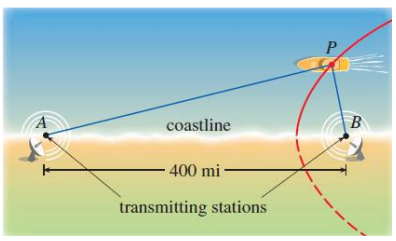
IV. Identify the type of graph (circle, ellipse, parabola, hyperbola, line, 2 intersecting lines, 2 parallel lines, point, empty set) given the general second degree equation.

1. $4x^2 + 9y^2 - 8x + 36y + 40 = 0$
2. $4x^2 - 9y^2 - 8x - 36y - 68 = 0$
3. $4x^2 - 8x - 32 = 0$
4. $4x^2 + 9y^2 - 8x + 36y + 4 = 0$
5. $4x^2 - 9y^2 - 8x - 36y - 32 = 0$
6. $y^2 - 4x + 4y + 8 = 0$
7. $4x^2 + 9y^2 - 8x + 36y + 76 = 0$
8. $9y^2 + 36y + 36 = 0$
9. $x^2 + y^2 - 2x + 4y - 31 = 0$
10. $4x^2 - 9y^2 - 8x - 36y + 4 = 0$

V. Solve the following problems.



1. **Parabola**. A cross –section of a parabolic reflector is shown in the figure. The bulb is located at the focus and the opening at the focus is 10 cm.
   a. Find an equation of the parabola.
   b. Find the diameter CD of the opening, 11 cm from the vertex.
2. **Ellipse**. The point in a lunar orbit nearest the surface of the moon is called *perilune* and the point farthest from the surface is called *apolune*. The *Apollo 11* spacecaft was placed in an elliptical lunar orbit with perilune altitude 110 km and apolune altitude 314 km (above the surface of the moon). Find an equation of the ellipse if the radius of the moon is 1728 and the center of the moon is at one focus.
3. **Hyperbola**. The LORAN (LOng RAnge Navigation) radio navigation system was widely used until the 1990s, when it was superseded by the GPS system. In the LORAN system, two radio stations located at A and B transmit simultaneous signals to a ship or an aircraft located at P. The onboard computer converts the time difference in receiving these signals into a distance difference PA – PB, and this, according to the definition of the hyperbola, locates the ship or aircraft on one brance of a hyperbola (see figure).



Suppose that station B is located 400 mi due east of station A on a coastline. A ship received the signal from B 1200 microseconds (μs) before it received the signal from A.
   a. Assuming that radio signals travel at a speed of 980 ft/μs, find an equation of the hyperbola on which the ship lies.
   b. If the ship is due north of B, how far off the coastline is the ship?

4. Find the length of a latus rectum of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
5. Find the distance between the directrices of the hyperbola $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$.
6. A hyperbola has a vertex at (4, 2), a corresponding focus at (4, 4), and a corresponding directrix the line y = 1. Find the center of the hyperbola.
7. Find the standard equation for the set of all points in the Cartesian Plane the absolute value of the difference of whose distances from the points (-3, 1) and (7, 1) is 6.
8. The vertices and foci of the ellipse $3x^2 + 4y^2 - 12 = 0$ are the foci and vertices, respectively, of a hyperbola. Find the standard equation of the hyperbola.
9. The vertex and directrix of the parabola $x^2 = 4y$ are, respectively, a covertex and principal axis of a hyperbola whose tranverse axis is as long as the latus rectum of the parabola. Find the standard equation of the hyperbola.
10. The vertices and covertices of a hyperbola are the covertices and foci, respectively, of another hyperbola. If the two hyperbolas have the same eccentricity, find the eccentricity.